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## **Brazil (Federative Republic of)**

### **RAIN ATTENUATION TIME SERIES SYNTHESIZERS FOR TERRESTRIAL LINKS**

#### **TESTING THE GAMMA DISTRIBUTION AND $\beta$ ESTIMATION**

## **1 Introduction**

Recommendation ITU-R P.1853 describes the use of synthesizers aimed at the generation of scintillations time series and rain attenuation time series. The rain attenuation time series synthesizers – originally based on the approach presented by Maseng and Bakken [1] - were tested, improved and validated for temperate European and tropical climates [2], [3], [4], [5], [6].

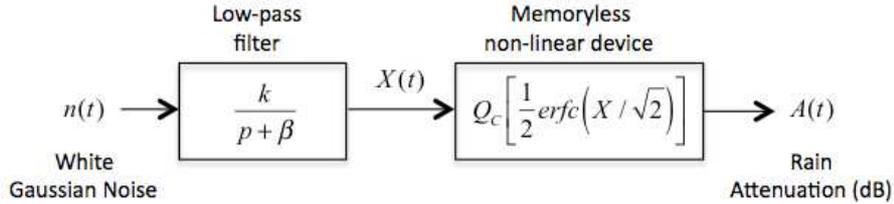
In this document, the use of rain attenuation time series synthesizers for terrestrial links is examined using experimental data obtained from five links at a tropical site [7]. Following the observation that the Gamma distribution better describes the complementary cumulative distribution function (CCDF) of the measured rain attenuation, a Gamma synthesizer was developed and the results are compared with those obtained with the EMB model. Also, empirical expressions to estimate the synthesizers dynamics parameter  $\beta$ , as a function of path length are obtained, for both the EMB and the Gamma models.

## **2 Gamma Synthesizer**

The Gamma model is a stochastic model of rain attenuation that generates random processes with specified first and second order statistics. It assumes the same basic hypothesis of the lognormal MB and EMB models, that rain attenuation may be synthesized from a first order Gaussian stationary Markov process. The block diagram of the Gamma synthesizer is presented in Fig. 1.

FIGURE 1

**Block diagram of the Gamma model.**



The model requires three parameters,  $b$ ,  $c$  and  $\beta$ . The first two are the scale and shape parameters of the Gamma distribution, which are derived by curve fitting from the long-term CCDF of rain attenuation. The dynamics parameter  $\beta$  is derived from the autocovariance function of the rain attenuation time series. The rain attenuation  $A(t)$  is obtained converting a Gaussian colored noise  $X$  through a memoryless nonlinear transformation

$$A_{RAIN} = Q_C \left[ 1/2 \operatorname{erfc} \left( X / \sqrt{2} \right) \right] \quad (1)$$

QC is that complementary quantile function, which does not have analytical expression for the Gamma distribution. A numerically implementation was developed using Matlab routines made available by the WAFO (Wave Analysis for Fatigue and Oceanography) group [8].

The experimental measurements of rain attenuation [7] were performed in five line-of-sight terrestrial links located in São Paulo, Brazil, for periods between one and two years depending on the link. The links operation frequencies and path lengths are given in Table 1.

TABLE 1

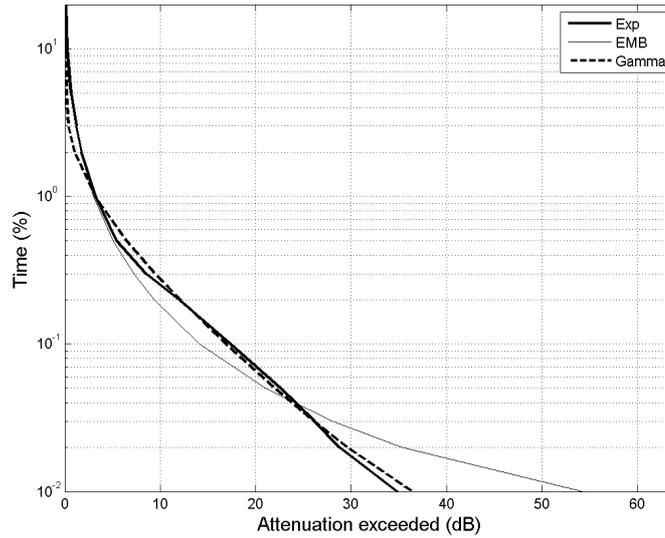
**Link characteristics,  $\beta$  parameters and r.m.s. errors**

Link	Path length (km)	Frequency (GHz)	Polarization
Bradesco	12.8	14.55	V
Cenesp	12.8	14.55	H
Scania	18.4	14.50	V
Barueri	21.7	14.53	V
Parana	43.0	14.52	H

The required parameters for the EMB and Gamma models were retrieved from the experimental data collected at each link. Both models were then used to synthesize 10 years of rain attenuation time series for all links. Statistics of rain attenuation and fade slope of the synthesized time series were obtained. The r.m.s. errors relative to the corresponding statistics of experimental data were determined, using the test variables recommended by the ITU-R [8].

Fig. 2 shows the rain attenuation CCDFs for the Bradesco link, obtained from the time series synthesized by the two models and the experimental data.

FIGURE 2  
Rain attenuation CCDF of synthesized and experimental data - Bradesco link.



For low values of attenuation, both the EMB and Gamma synthesized CCDFs are close to the experimental curve. However, for the high values of attenuation, which are particularly relevant for designing fade mitigation systems, the Gamma CCDF provides a much better approximation of the experimental results. Similar behaviour was verified for the other four links.

Table 2 shows the r.m.s. values of the test variable [9] for the rain attenuation CCDF in a range of time percentages between 0.01% and 1%. The results can be considered very good. Also shown are the r.m.s. errors for the rain fade slope CCDF at attenuation thresholds from 3 and 30 dB. The fade slope prediction is less accurate, but the results are comparable to those obtained for satellite links [10].

TABLE 2  
Link characteristics,  $\beta$  parameters and r.m.s. errors

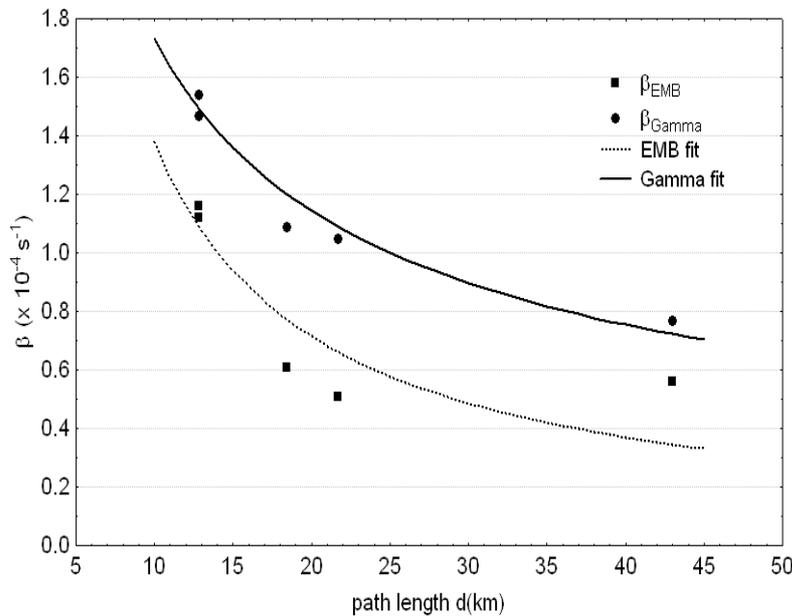
Link	Model	$\beta (s^{-1})$	Attenuation r.m.s. error	Fade slope r.m.s. error
Bradesco	EMB	$1.16 \times 10^{-4}$	0.21	0.80
	Gamma	$1.47 \times 10^{-4}$	0.07	0.72
Cenesp	EMB	$1.12 \times 10^{-4}$	0.16	0.94
	Gamma	$1.54 \times 10^{-4}$	0.05	0.80
Scania	EMB	$0.61 \times 10^{-4}$	0.07	0.82
	Gamma	$1.09 \times 10^{-4}$	0.10	0.78
Barueri	EMB	$0.51 \times 10^{-4}$	0.13	0.95

	Gamma	$1.05 \times 10^{-4}$	0.08	0.74
Parana	EMB	$0.56 \times 10^{-4}$	0.14	1.03
	Gamma	$0.77 \times 10^{-4}$	0.05	0.91

### 3 $\beta$ parameter estimation

The main difficulty in the practical application of these models is the determination of the synthesizer dynamics parameter  $\beta$  when no experimental time series is available. For terrestrial links it was observed that there is significant correlation between the path length  $d$ (km) and the values of  $\beta$  particularly for the Gamma model, as shown in Fig.3.

FIGURE 3  
 $\beta$  parameter fittings.



The following empirical expressions were derived to estimate  $\beta$  (s-1).

$$\beta_{EMB} = 12.3 d^{-0.95} \times 10^{-4} \quad (2)$$

$$\beta_{Gamma} = 6.9 d^{-0.6} \times 10^{-4} \quad (3)$$

### 4 Conclusion

The results indicate that Gamma model is particularly good for synthesizing the cumulative distribution of rain attenuation in terrestrial links. It also improves the results for fade slope statistics at high levels of rain attenuation. This is important for the design and optimization of FMTs for systems operating in tropical regions, which are subject to high rainfall rates. Empirical expressions for estimating the dynamics parameter  $\beta$  as a function of the path length were obtained, that allow the synthesizer implementation without the need of experimental data.

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